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36 e

50 b

38 d

40 f

6.2 U-Substitution

If F is such that $F'(x) = f(x)$ then,

$$\int f(x)\,dx = F(x) + C$$ indefinite integral (a family of curves)

$$\int_a^b f(x)\,dx = F(b) - F(a)$$ definite integral (a number)

$$F(b) + C - [F(a) + C]$$

$$F(b) - F(a)$$

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Properties of Indefinite Integrals

$$\int k f(x) dx = k \int f(x)\, dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x))\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

Power Formulas

$$\int u^n\, du = \frac{u^{n+1}}{n+1} + C \text{ when } n \neq -1$$

$$\int u^{-1}\, du = \int \frac{1}{u}\, du = \ln|u| + C$$

(see Example 2)

Trigonometric Formulas

$$\int \cos u\, du = \sin u + C$$

$$\int \sin u\, du = -\cos u + C$$

$$\int \sec^2 u\, du = \tan u + C$$

$$\int \csc^2 u\, du = -\cot u + C$$

$$\int \sec u \tan u\, du = \sec u + C$$

$$\int \csc u \cot u\, du = -\csc u + C$$

Exponential and Logarithmic Formulas

$$\int e^u\, du = e^u + C$$

$$\int a^u\, du = \frac{a^u}{\ln a} + C$$

☆ $$\int \ln u\, du = u \ln u - u + C \quad \text{(See Example 2)}$$

☆ $$\int \log_a u\, du = \int \frac{\ln u}{\ln a}\, du = \frac{u \ln u - u}{\ln a} + C$$

Why the dx?

Let $f(x) = x^3+1$, let $u = x^3$. Find each in terms of x.

① $\int f(x)\,dx = \int (x^3+1)\,dx = \frac{x^4}{4} + x + C$

② $\int f(u)\,du = \int (u^3+1)\,du = \frac{u^4}{4} + u + C = \frac{x^{12}}{4} + x^3 + C$

③ $\int f(u)\,dx = \int (u^3+1)\,dx = \int (x^9+1)\,dx = \frac{x^{10}}{10} + x + C$

U-Substitution

① $\int \sin x \, e^{\cos x} \, dx$

Let $u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

Rewrite in terms of u.

$$-\int e^u \, du = -e^u + C = -e^{\cos x} + C$$

② $\int x^2 \sqrt{5+2x^3} \, dx$

$u = 5+2x^3$

$du = 6x^2 \, dx$

$\frac{1}{6} du = x^2 \, dx$

$$\frac{1}{6}\int \sqrt{u} \, du$$

$$= \frac{1}{6}\int u^{1/2} \, du = \frac{1}{6} \cdot \frac{u^{3/2}}{\frac{2}{3}} + C$$

$$= \frac{1}{9} u^{3/2} + C = \frac{1}{9}(5+2x^3)^{3/2} + C$$

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